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Kuo-Jen Hwang[a]; Wei-Tang Chu[a]; Wei-Ming Lu[b]
[a] Department of Chemical Engineering, Tamkang University, R.O.C. [b] Department of Chemical Engineering, National Taiwan University, R.O.C.

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A METHOD TO DETERMINE THE CAKE PROPERTIES IN CENTRIFUGAL DEWATERING

Kuo-Jen Hwang,[1,]* Wei-Tang Chu,[1] and Wei-Ming Lu[2]

[1]Department of Chemical Engineering, Tamkang University, Tamsui, Taipei Hsien, Taiwan 25137, R.O.C.
[2]Department of Chemical Engineering, National Taiwan University, Taipei, Taiwan 10617, R.O.C.

ABSTRACT

A method for determining the cake properties in centrifugal dewatering is proposed in this study. By substituting experimental data of cake saturation versus time into the proposed procedures, the values of capillary pressure and filter cake permeability under various cake saturations can be estimated. Two kinds of particulate samples, $CaCO_3$ and polystyrene powder, were used in the centrifugal dewatering experiments. The results show that both cakes exist in their funicular state for most times of dewatering. The value of capillary pressure increases while the permeability of cake decreases during the course of centrifugal dewatering. The regression functions relating the capillary pressure and permeability to the cake saturation are in agreement with the empirical equations used in previous work. A program was also designed to simulate the variation of cake saturation during centrifugal dewatering. A good agreement between the simulated results and the experimen-

*Corresponding author. Fax: 886-2-26209887; E-mail: kjhwang@mail.tku.edu.tw

tal data was obtained. The optimum operating condition for a given sample can be found by using the simulation method.

Key Words: Centrifugal dewatering; Cake properties; Saturation; Capillary pressure; Centrifuge

INTRODUCTION

Centrifugal dewatering is an efficient method for solid-liquid separation in many industrial processes. Because the driving pressure produced by a centrifuge is far larger than that by a pump in most conditions, centrifugal dewatering has many advantages, e.g., high dewatering rate and low moisture content of cake. Although the centrifuge has been widely used in many practices, many operating factors still need to be understood more clearly before the optimum operating condition can be determined for a given product.

The water contained in a packed particulate bed has 3 forms: the surface water, the structure water, and the chemical water (1). The surface water lies on the surface of particles, and it can be drained more easily. The structure water exists in the capillary structure of individual particles, while the chemical water is held in the chemical composition. In general, only the surface water must be taken into account for centrifugal dewatering.

The capillary pressure in a cake plays a major role in the efficiency of a centrifugal dewatering operation. The capillary pressure is determined by several factors, such as the size and shape of particles, structure of the cake, and the liquid content in the cake. For cakes formed by a given particulate sample, the liquid content is the most important factor. The states of liquid contained in a particulate bed can be divided into the capillary state, the funicular state, and the pendular state (2). The relationships between capillary pressure and cake saturation in these states are shown in Fig.1. In the capillary state, the particulate bed approaches the saturation state, and the capillary pressure is trifling in such a condition. If some liquid in the micropore between particles has been drained off and the particle surfaces are still covered with a liquid film, the state is funicular. The capillary pressure at a funicular state is far larger than that at capillary state but it varies slightly with the saturation of particulate bed. In the pendular state, the liquid only exists at the contact point between particles, and the capillary pressure increases rapidly as the bed saturation decreases. When the saturation of a particulate bed decreases to a critical value, the centrifugal pressure will become insufficient to overcome the huge capillary pressure. As a result, the liquid in the cake cannot be further removed, and the cake saturation under such a condition is referred to as "equilibrium saturation." Although the capillary pressures under various states of cake saturation can be measured, the experiments are very difficult and time consuming to perform.

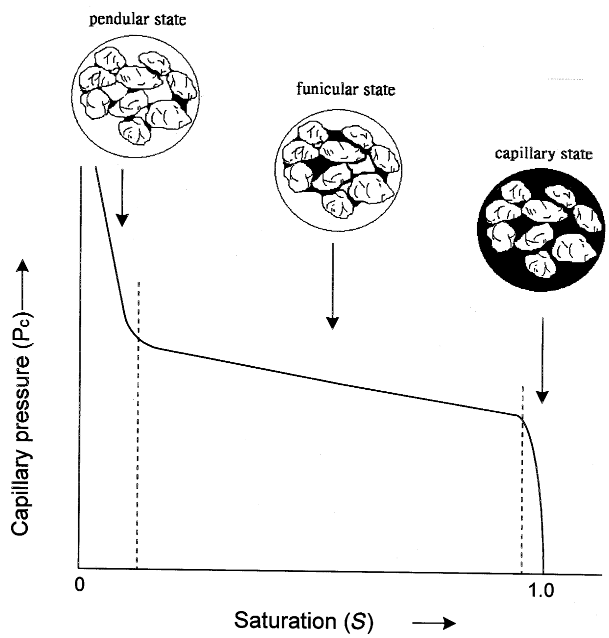

Figure 1. Relationships between capillary pressure and saturation of cake in 3 states.

Several theoretical models, such as film flow, capillary, and network models, for centrifugal or gravity dewatering from porous particulate beds have been proposed (3). Under the film flow model, a certain amount of liquid will be left around the particle surfaces as the bulk liquid level falls during drainage. The retained liquid then drains as a falling film over the particle surface. Nenniger and Storrow (4) indicated that the presence of the liquid film reduced the dehydration rate. An analogue from the film flowed over a flat plate has also been developed by Nenniger and Storrow (4). Shirato, Murase, and Mori (5) assumed that the pore structure in a particulate bed is composed of a number of capillaries with different diameters, and they obtained a relationship between the average saturation and time by taking the effects of capillary and film flow into account. However, the equilibrium saturation of cake under various conditions should be measured in experiments and substituted into their model. Wakeman and Vince (3,6) developed a mathematical-physical model to predict the transient saturation profiles in a particulate bed and the overall drainage rate. In their model, the centrifuge cakes are considered a moist particulate bed, and the empirical relationships between the capillary pressure, relative permeability, and reduced saturation are adopted. Based on Darcy's law and the continuity equation of the bed, the distribution of cake saturation during centrifugal dewatering can be simulated.

Because the dewatering properties of a cake are difficult to obtain from theoretical derivations or experimental measurements, many time-consuming experiments must be carried out if the performance of a centrifugal dewatering for a specific sample is to be understood. Therefore, a simple calculation method for estimating the cake permeability and the capillary pressure in a cake is proposed in this article. In the calculation procedures, only the values of average saturation of cake versus time for a given operating condition, which can be obtained readily, are required. Furthermore, a numerical program has been developed to simulate the variation of average cake saturation values during centrifugal dewatering operations. The optimum operating condition for a specific sample can also be found by the proposed method.

THEORY

Flow of Liquid in Cake

A liquid flowing through a filter cake can be described using Darcy's law; that is

$$q = \frac{K}{\mu}\frac{(P - P_c)}{L} \tag{1}$$

where q is the flow rate of liquid; K is the permeability of the cake; μ is the viscosity of the liquid; L is the cake thickness; and P and P_c are the centrifugal pressure that drops through the cake and the capillary pressure in the cake, respectively.

Because most of cake compression of the particulate samples used in this study occurred during the filtration period, the cake in the dewatering period can be considered incompressible. Therefore, the local pressure drop in the one-dimensional dewatering system shown in Fig. 2 can be expressed as (6)

$$\frac{dP}{dr} = \rho r \omega^2 \tag{2}$$

where ρ is the density of liquid, while ω is the angular velocity. Eq. (2) can be integrated with respect to r to give

$$P = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) \tag{3}$$

where P is the pressure drop between r_1 and r_2. The surface of cake particles be assumed to be covered with a thin film of liquid during dewatering and the liquid phase can be considered a continuum. Thus, the dewatering takes place in the cap-

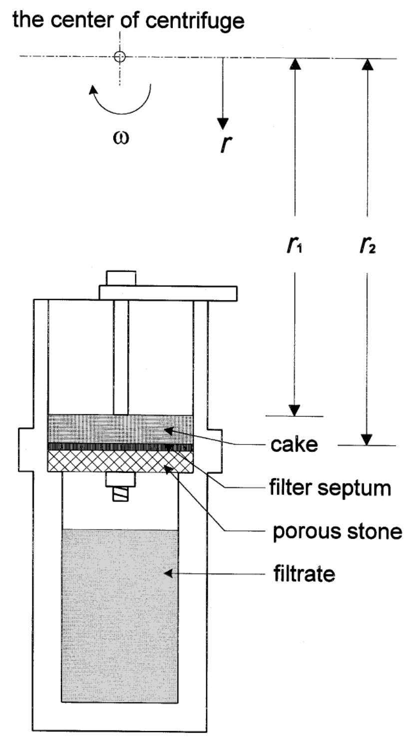

Figure 2. A one-dimensional centrifugal dewatering system.

illary and funicular states of cake, and Eq. (3) is valid for estimating the pressure drop through the cake during a centrifugal dewatering operation.

If the cake is thin enough and composed of fairly coarse particles, the equilibrium saturation will be distributed uniformly throughout the whole cake (3). In thick cakes, the saturation is a function of *r*. If the end effects of the area are neglected, the average saturation can be given by

$$\overline{S} = \frac{1}{r_2 - r_1}\int_{r_1}^{r_2} S\,dr \tag{4}$$

A fundamental equation that relates the average saturation to capillary pressure was derived by Ayappa et al. (7) as

$$\overline{S} = \frac{r_1 + r_2}{2r_2 P_{c1}}\int_{0}^{P_{c1}} \frac{S}{(1 - BP_c/P_{c1})^{1/2}}\,dP_c \tag{5}$$

where P_{c1} is the capillary pressure at the septum surface and $B = 1 - r_1^2/r_2^2$. For the case of thin samples, the approximation for the local saturation can be given by (7)

$$S_1 = \overline{S} + P_{c1}\frac{d\overline{S}}{dP_{c1}} \tag{6}$$

where S_1 is the saturation at the septum surface. Thus, the relationship between capillary pressure and saturation at the septum surface can be obtained from Eq. (6) once the average saturation of cake under various rotational speeds is measured. When the rotational speed is high, the last term in Eq. (6) is small and can be neglected. In such a condition, the distribution of saturation in a thin cake can be considered uniform.

A mass balance on the liquid in a thin layer of filter cake gives (3,6)

$$\frac{\partial q}{\partial r} = -\varepsilon\frac{\partial\overline{S}}{\partial t} \tag{7}$$

where ε is the porosity of cake. This equation represents the relationship between the variation of cake saturation and the dewatering rate.

Estimation of Permeability and Capillary Pressure in Cake

To obtain the relationships among the permeability, capillary pressure, and saturation of a filter cake, the experimental data of average saturation of a cake versus dewatering time were substituted into the calculation procedure described below.

1. At the beginning of dewatering (i.e., the end of the filtration period), the flow rate of liquid and the pressure drop through the formed filter cake can be measured experimentally. Because the cake is in saturation ($\overline{S}$ = 1), the capillary pressure is equal to zero. The filtration pressure can be calculated using Eq. (3), and the permeability can be calculated using Eq. (1).
2. By measuring liquid content after each time increment, the average saturation of the cake can be known. Using the measured value of cake porosity, the dewatering rate can be calculated from Eq. (7). The average dewatering rate in the time interval can be obtained by using a centered-difference scheme (8). By substituting the average dewatering rate and the capillary pressure obtained at the previous time interval into Eq. (1), the cake permeability at the time can be estimated.
3. Substituting the values of dewatering rate and permeability of cake at the time into Eq. (1), the capillary pressure at the time can be estimated.

4. Steps 2 and 3 should be repeated until the saturation of cake approaches its equilibrium value. Then the variations of capillary pressure and permeability of cake for the entire path of centrifugal dewatering can be obtained.

Simulation of Centrifugal Dewatering

Once the relationships among the permeability, the capillary pressure, and the saturation of a cake are produced from the calculation procedure described, the dewatering curve can be simulated. The main procedure is described as follows:

1. Because the cake is saturated at the beginning of dewatering, the capillary pressure is equal to zero, and the dewatering rate can be given by substituting the calculated value of permeability into Eq. (1).
2. After a time increment, the values of capillary pressure and permeability of cake can be estimated from the previous condition. Then, the dewatering rate can be calculated from Eq. (1), and cake saturation can be recalculated accordingly.
3. Step 2 should be repeated until the saturation of cake remains constant. Then the variation of cake saturation during the course of centrifugal dewatering can be obtained.

EXPERIMENTAL

Batchwise centrifugal dewatering procedures were carried out using a centrifuge like that depicted in Fig. 3. The filter chamber was made from a clear acrylic to facilitate observation. The distance from the center of the centrifuge to the surface of filter septum was 0.164 m, and the rotation speed of the centrifuge ranged from 300 to 1300 rpm, which resulted in a maximum centrifugal effect of $Z = 300$.

Two particulate samples were used in experiments: $CaCO_3$ particles with a density of 2700 kg/m^3 and diameters ranging from 3 to 30 μm and spherical polystyrene of density 1060 kg/m^3 and diameters ranging from 50 to 800 μm. In this study, a cake approximately 0.005–0.01 m thick was obtained. Therefore, the distribution of saturation in the cake can be considered to be uniform.

Before the experiment, the particles were weighed and suspended in water by a mixer to prepare a slurry. When a certain amount of slurry was poured carefully into the filter chamber, the centrifuge was turned on and a predetermined rotational speed was reached as soon as possible. The image of the filter chamber was caught with a stroboscope and was recorded with a CCD camera during experiments. The slurry height, the cake thickness, and the filtrate volume were

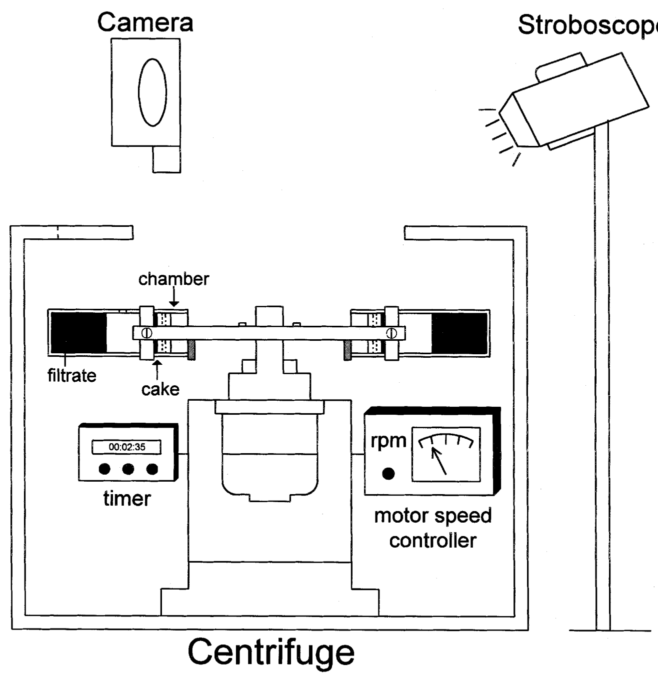

Figure 3. Schematic diagram of centrifuge used in dewatering experiments.

observed through the clear acrylic wall of the filter chamber. Therefore, the volumes of cake and filtrate were observed, and the average porosity of the filter cake could be estimated accordingly. The saturation of cake was measured either by weighing the wet and dry filter cake or measuring the filtrate volume.

RESULTS AND DISCUSSION

Figures 4 and 5 show the capillary pressure and permeability of the cake formed by polystyrene particles under various saturations. The data shown in these figures were obtained by substituting the experimental data of cake saturation at various dewatering times under 5 rotational speeds into the calculation procedures previously described. Although these data were generated under different centrifugal conditions, they followed the same regression curve. This implies that the cake structures are the same under these operating conditions. Furthermore, from the rapid increase of capillary pressure at the region of where S approaches 1, one can conclude that the capillary state of the cake disappears very quickly at

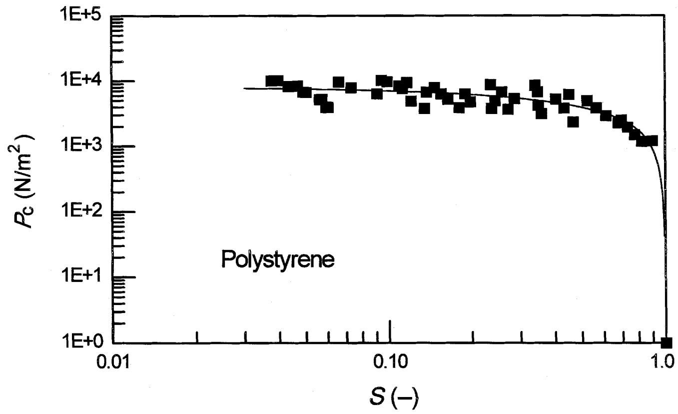

Figure 4. Calculated capillary pressures under various saturations of polystyrene cake.

the beginning of dewatering and that most dewatering operates in the funicular state of the cake. This result also demonstrates that the assumption of the continuous phase of the liquid is valid.

The relationship between permeability and saturation of polystyrene cake shown in Fig. 5 indicated that the value of permeability decreases as the cake saturation decreases. This implies that the dewatering becomes more difficult when the cake saturation decreases gradually during the centrifugal dewatering operation. Moreover, the relationship between K and S can be described by the follow-

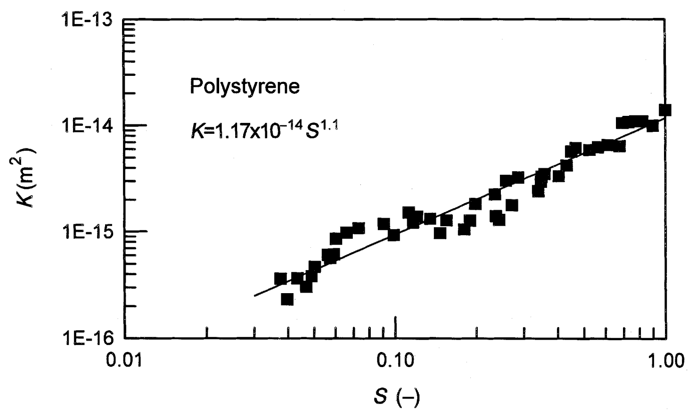

Figure 5. Calculated permeability of polystyrene cake at different saturations.

ing empirical correlation (3):

$$K = K_o \bar{S}_R^{(2+3\lambda)/\lambda} \tag{8}$$

where K_o is the permeability of cake at saturation, λ is a constant, and $[\bar{S}]_R$ is the relative saturation of cake and is defined as

$$\bar{S}_R = \frac{\bar{S} - \bar{S}_\infty}{1 - \bar{S}_\infty} \tag{9}$$

in which $[\bar{S}]_\infty$ is the equilibrium saturation of cake. The results of this study agree with those from the function form depicted in Eq. (8).

The capillary pressure and permeability of $CaCO_3$ cake under various saturations are shown in Figs. 6 and 7. Figure 6 shows that most of the dewatering occurs in the funicular state of cake, and the same regression curve can be given for different rotation speeds. Furthermore, the results shown in Fig. 7 agree with the data obtained from regression functions obtained from previous experimental results.

Figure 8 shows the comparison of $CaCO_3$ cake saturation between simulated results and experimental data during centrifugal dewatering under various rotation speeds. From the curves shown in the figure, one can notice that saturation of cake decreases quickly during the initial stage of dewatering due to the large permeability and small capillary pressure in this period. However, if the state of cake saturation changes to funicular, the dewatering rate decreases due to the increased capillary pressure and decreased cake permeability, and the saturation of cake gradually approaches the equilibrium value under such a condition. The simulated results agree fairly well with the experimental data except near the equilibrium saturation time. Moreover, the increase of centrifuge rotation speed may

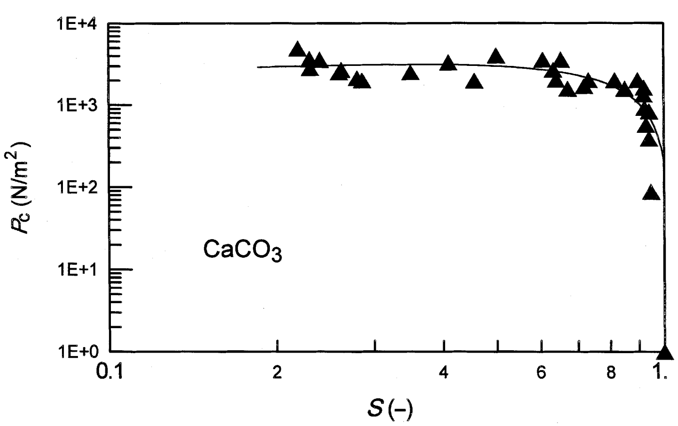

Figure 6. Calculated capillary pressure under various saturations of $CaCO_3$ cake.

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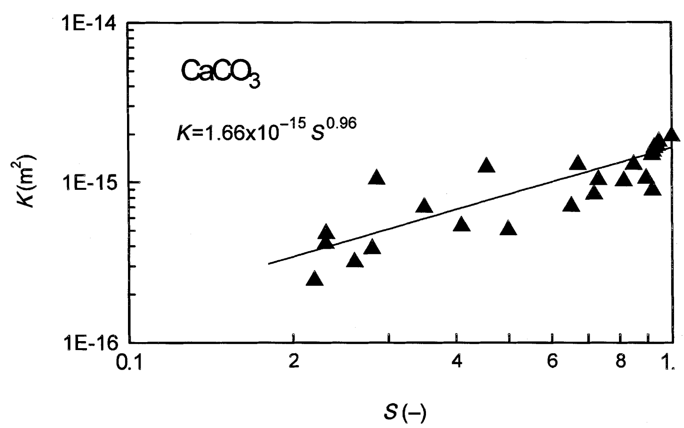

Figure 7. Calculated permeability of $CaCO_3$ cake at different saturations.

improve the efficiency of dewatering, but this effect becomes less obvious at high rotating speed.

Figure 9 shows the simulated dewatering curves under 3 different centrifugal pressures. From this figure, one can see that the equilibrium saturation under $P = 5000$ N/m^2 is smaller than that under $P = 2000$ N/m^2 due to the increase of

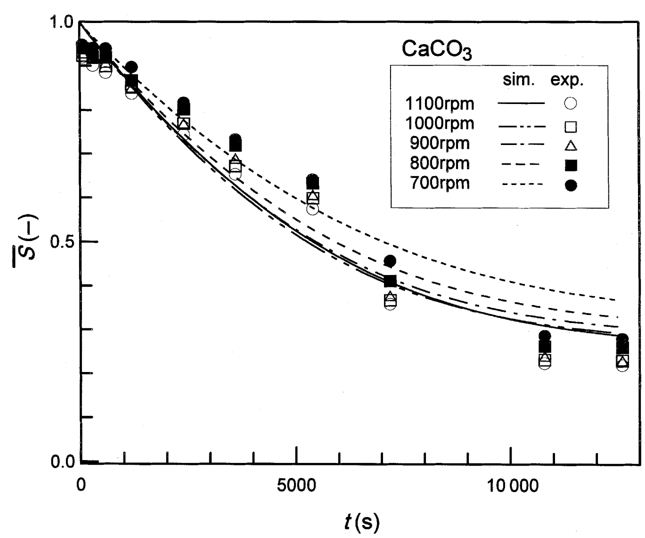

Figure 8. Comparison of saturation between simulated results and experimental data during centrifugal dewatering.

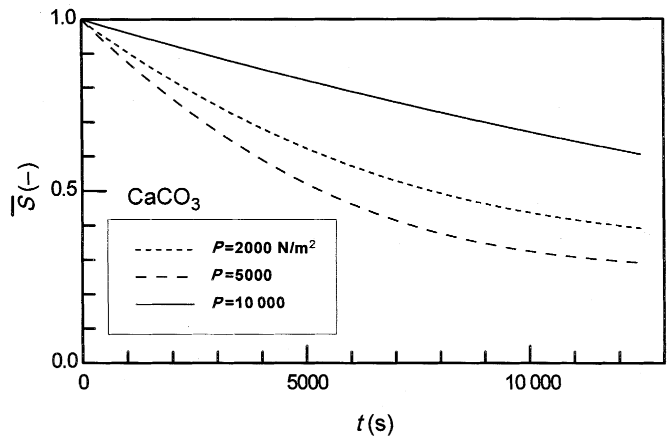

Figure 9. Simulated saturation of $CaCO_3$ cakes during centrifugal dewatering at 3 different centrifugal pressures.

dewatering driving force. However, the equilibrium saturation under $P = 1.0 \times 10^4$ N/m^2 is unexpectedly higher than those of $P = 2.0 \times 10^3$ and 5.0×10^3 N/m^2. Because a large centrifugal pressure results in a larger dewatering rate and lower saturation at the beginning of operation, the rapid increase of capillary pressure and decrease of permeability of cake causes dewatering of the filter cake to be more difficult at the later operation period.

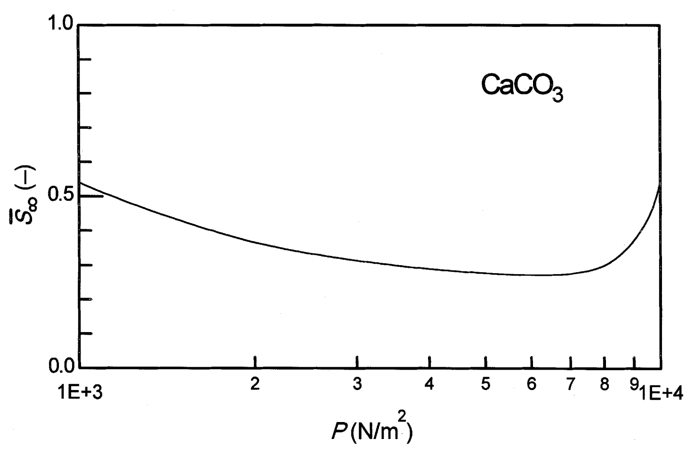

Figure 10. Simulated equilibrium saturation of $CaCO_3$ cakes under various centrifugal pressures.

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To understand the effect of centrifugal pressure on the equilibrium saturation of cake, the dewatering curves of $CaCO_3$ cake under various conditions were simulated, and the values of equilibrium saturation are shown in Fig. 10. When the centrifugal pressure increases from 1000 to 6500 N/m^2, the value of equilibrium saturation decreases with the increase of centrifugal pressure (rotational speed). Therefore, increasing the rotation speed can effectively decrease the liquid content in the cake within these operating conditions. However, the equilibrium saturation increases as the centrifugal pressure increases from 6.5×10^3 to 1.0×10^4 N/m^2. These results imply that the optimum operating condition for a given sample can be found by the simulation method proposed in this study.

CONCLUSION

By substituting the experimental data of cake saturation into the proposed procedures, the values of capillary pressure and permeability of filter cake were estimated from centrifugal dewatering operations. The values of capillary pressure rapidly increased at the beginning of dewatering, while the permeability of cake decreased during centrifugal dewatering. For the samples used in this study, the operations proceeded in the funicular state of cake for most of the dewatering period. The regression function relating the capillary pressure to the saturation of particulate bed agreed with the empirical equation used in previous work. A program was developed to simulate the variation of saturation during the course of centrifugal dewatering. The simulated results agreed fairly well with the experimental data. This result demonstrates the accuracy of the calculation by the proposed method. The optimum operating condition for a specific sample could be found using the simulation method.

NOMENCLATURE

B	a parameter in Eq. (5) $(=1 - r_1^2/r_2^2)$ $(-)$
K	permeability of cake (m^2)
K_o	permeability of cake at saturation (m^2)
L	cake thickness (m)
P	pressure drop through the cake (N/m^2)
P_c	capillary pressure (N/m^2)
P_{c1}	capillary pressure at the septum surface (N/m^2)
q	flow rate of water in cake ($m^3/m^2 \cdot s$)
r	distance from the center of centrifuge (m)
r_1	distance from the center of centrifuge to the cake surface (m)
r_2	distance from the center of centrifuge to the surface of filter septum (m)

S	local saturation of cake (−)
$[\bar{S}]$	average saturation of cake (−)
$[\bar{S}]_R$	relative saturation of cake defined in Eq. (9) (−)
$[\bar{S}]_8$	equilibrium saturation of cake (−)
t	time (s)
Z	centrifugal effect (the ratio of centrifugal force to gravity force) (−)

Greek Letters

ε	porosity of cake (−)
λ	empirical constant used in Eq. (8) (−)
μ	viscosity of fluid (kg/s·m)
ρ	density of liquid (kg/m^2)
ω	angular velocity (rad/s)

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